

STUDY OF PARTICLE ADHESION IN A
FLUIDIZED BED ON A COLD MODEL

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Until now the phenomenon of particle adhesion in a fluidized bed during high-temperature treatment of loose materials (for firing clay into chamotte, for firing dolomite, for reducing iron-ore charges, etc.), has not yet been explored adequately. The purpose of this study is to establish the general laws governing the adhesion phenomenon.

Based on an analysis of surface and volume forces acting on a particle in a fluidized bed with a liquid phase around, a general functional relation is obtained between the maximum liquid content in a bed and the hydrodynamics of fluidization as well as the physical properties of the solid and the liquid phase.

The adhesion phenomenon was studied in experiments with a cold model. The tests were performed with quartz sand in five fractions: 0.5-0.7 mm, 1.0-1.25 mm, 1.25-1.6 mm, 2.0-2.5 mm, and 3.0-5.0 mm as well as with 2.0-3.0 mm limestone. As the liquid phase the authors used glycerine, ethylene glycol, and sunflower oil. The tests covered a wide range of flow velocities, while the fluidization number was varied from 1 to 6. In order to reveal the effect of a distributor grid on the stability limit of a fluidized bed, the hydraulic resistance of the grid was varied in the tests: the maximum grid resistance was made equal to five times the bed resistance.

The semiempirical relation between the relative liquid content q in a fluidized bed of solid material at the stability limit is

$$q = 2.76 \cdot 10^{-3} \left(\frac{d^2 \gamma_s}{\sigma} \right)^{0.65} (m-1)^n \left(\frac{\Delta p_b}{\Delta p_g} \right)^{-0.585}$$

where $n = 1.13d^{0.102}$ with the diameter of fluidized material particles d ; the density of the solid material γ_s ; the surface tension of the liquid σ ; the fluidization number m ; and the hydraulic resistances of bed and grid Δp_b , Δp_g , respectively.

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INTERPHASE HEAT TRANSFER IN A FLUIDIZATION
BED WITH HEAT SOURCES

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The heat transfer between the dense phase and the bubble layer in a fluidization bed is analyzed for the case when one of the phases contains heat sources (chemical or nuclear reactors, inductive heaters of particles, etc.). The heat-transfer rate is determined from the magnitude of the effective interphase transfer coefficient β (sec^{-1}) analogous to the interphase mass-transfer coefficient. The equation of one-dimensional steady-state temperature distribution in each phase is solved for the cases of ideal displacement and ideal mixing in the dense phase. The following distributions of heat sources in each phase are considered: an arbitrary power distribution of sources throughout the heat emitting phase, a corresponding uniform distribution, and a linearized exponential temperature-dependence of the power of heat sources (occurrence of zeroth-order exothermal and endothermal processes).

For the last case, with ideal mixing in the dense phase, the temperature distribution is

$$\bar{\theta} = \bar{P}_0 / \left\{ 1 + \frac{N_1}{N_2} [1 - \exp(-N_2) - \bar{P}_0] \right\};$$

$$\bar{\psi} = \bar{P}_0 [1 - \exp(-N_2 y)] / \left\{ 1 + \frac{N_1}{N_2} [1 - \exp(-N_2) - \bar{P}_0] \right\};$$

and when heat is generated in the bubbles

$$\bar{\theta} = \bar{P}_0 / [(N_2 - \bar{P}_0)(1 + N_1) - N_2 \Phi]; \quad \bar{\psi} = \bar{P}_0 \{1 - \exp[-(N_2 - \bar{P}_0)y]\} \left[1 + \frac{N_2}{(N_2 - \bar{P}_0)(1 + N_1) - N_2 \Phi} \right] / (N_2 - \bar{P}_0);$$

$$\Phi = 1 - \frac{1}{N_2 - \bar{P}_0} \{1 - \exp[-(N_2 - \bar{P}_0)]\}.$$

Here $\bar{\theta} = E(\theta^* - \theta_0)/R(\theta_0 + 273)^2$; $\bar{\psi} = E(\psi^* - \theta_0)/R(\theta_0 + 273)^2$ are the dimensionless temperatures of the dense phase and of the bubbles, respectively; $\bar{P}_0 = EQH/Rw_g c_g \gamma_g \varepsilon_0 (\theta_0 + 273)^2$ is the Pomerantsev number; θ^* , ψ^* , θ_0 are the temperature of gas in the dense phase, in the bubbles, and at the bed entrance, respectively; $N_1 = \beta H/w_0 \varepsilon_0 (1 - \varepsilon_b)$; $N_2 = \beta H/(w_g - w_0)$ are the parameters of interphase transfer for each of the two phases; w_0 is the gas velocity at the beginning of bubble formation; w_g is the linear velocity of the gas; ε_b is the volume fraction of the bed occupied by bubbles; ε_0 is the porosity of the dense phase; h , H are the height coordinate and the total bed height, respectively; $y = h/H$; E is the process activation energy; R is the gas constant; c_g , γ_g are the specific heat and the density of the particle material; Q is the thermal flux density; and W is the fluidization number.

For the case of constant heat generation in the dense phase, the temperature distributions in both phases are calculated along with the exit temperature during ideal displacement and mixing in the dense phase over a wide range of N_1 , W , and ε_b values. The results indicate a significant difference in the temperatures of the phases, even when the interphase transfer rate is high. During ideal mixing in the dense phase and at high values of N_1 the bubble temperature approaches the temperature of the dense phase only at one fifth of the total bed height. As ε_b and W are increased, the temperatures of both phases drop slightly while the exit temperature drops sharply. The difference between the phase temperatures in the upper bed regions exerts a considerable influence on the trend of the processes in a fluidization bed (especially of complex chemical processes, where the temperature distribution determines their feasibility and selectivity on an industrial scale).

EFFECT OF THE TEMPERATURE OF A CONCRETE
SPECIMEN ON ITS STRENGTH

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The increase in the mechanical strength of concrete test specimens, as a function of the temperature to which they are heated, is within the scope of several research programs. Experimental studies were made on this subject, to determine the effect of the temperature on the strength of a test specimen (Table 1). The weight ratios between the concrete components in the test specimens were 0.5:1:1:1.6:2.2 and the degree of cement hydration was 0.74.

Analysis of these results showed that the following relationships held between the strength of dry (R_d) and water saturated ($R_{w.s.}$) concretes and the temperature t for $t \in [293^\circ\text{K}, 368^\circ\text{K}]$:

$$10^{-4} \cdot R_d = 4851 - 0.63(t - 293),$$

$$10^{-4} \cdot R_{w.s.} = 3541 - 2.53(t - 293).$$

At a 0.05 significance level, an estimate based on the Fisher distribution has shown that the linearity hypothesis agrees with the authors' test data.

The tests performed here support the hypothesis that, within the given temperature range, the temperature and the humidity interdependently affect the strength of cured concrete.

TABLE 1

Specimens	Group	Specimen temperature $^\circ\text{K}$	Average strength of concrete, \bar{R} $\cdot 10^{-4} \text{ N/m}^2$	Unbiased estimate of the dispersion of strength values $S(\bar{R}) \cdot 10^{-8} (\text{N/m}^2)^2$
Dry	1	293	5017	4922
	2	308	4716	5937
	3	323	4668	13069
	4	338	4907	9126
	5	353	4766	8770
	6	368	4893	9126
Saturated with water	1	293	3494	6034
	2	308	3453	4878
	3	323	3593	12363
	4	338	3413	6379
	5	353	3438	3321
	6	368	3286	4043

Note. \bar{R} is the average based on the test results for 12 finned cubic specimens.

SOLVING THE DIFFUSION EQUATION FOR
AN N-COMPONENT SYSTEM

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UDC 539.219.3

The diffusion process in an n-component system is described by the following system of equations:

$$\frac{\partial c_i}{\partial t} = \sum_{j=1}^n D_{ij} \frac{\partial^2 c_i}{\partial x^2} \quad (i = 1, 2 \dots n). \quad (1)$$

The diffusion coefficients D_{ij} are assumed constant. The conditions under which Eqs. (1) are applicable have been formulated in [1]. Here the system is solved for the following initial and boundary conditions:

$$\begin{aligned} c_i(0, t) &= f_i(t), & t > 0, \\ c_i(x, 0) &= 0, & x > 0, \\ c_i(\infty, t) &= 0, & t > 0. \end{aligned} \quad (2)$$

The solution to the stated problem in transformed coordinates is

$$\bar{c}_i(x, p) = \sum_{j=1}^n A^j k_i^j \exp(x \kappa^j \sqrt{p}), \quad (3)$$

where κ_j are the roots of the equation

$$\begin{aligned} \det(D_{ij} \kappa^2 - \delta_{ij}) &= 0, \\ k_i^j &= (-1)^{i+1} \{ \det [D_{ij} (\kappa^j)^2 - \delta_{ij}] \} A^j, \quad (n, i). \end{aligned} \quad (4)$$

The meaning of (n, i) is that row n and column i in the determinant (4) are deleted; A^j are arbitrary constants found from the boundary conditions (2):

$$\sum_{j=1}^n A^j k_i^j = \bar{f}_i(p); \quad (5)$$

from which

$$A^j = \frac{\Delta_j}{\det(k_i^j)_1^n},$$

where determinant Δ_j has been derived from $\det(k_i^j)_1^n$ by replacing the j -th column with the right-hand sides of system (5).

The solution can be expressed in an explicit form for certain specific boundary conditions:

1. Diffusion from a constant source $c_i(0, t) = c_1^j$

$$c_i(x, t) = \sum_{j=1}^n \frac{\Delta_j}{\det(k_i^j)_1^n} \operatorname{erfc} \left(\frac{x \kappa^j}{2 \sqrt{t}} \right).$$

2. Concentration at the surface changes linearly with time $c_i(0, t) = \alpha_1 t$,

$$c_i(x, t) = \sum_{j=1}^n \frac{\Delta_j'}{\det(k_i^j)_1^n} \left\{ \left[t + \frac{x^2 (\kappa^j)^2}{2} \right] \operatorname{erfc} \frac{x \kappa^j}{2 \sqrt{t}} - x \kappa^j \sqrt{\frac{t}{\pi}} \exp \left[-\frac{x (\kappa^j)^2}{4t} \right] \right\},$$

where $\Delta_j' = p \Delta_j$ and does not contain p .

3. Concentration at the surface changes parabolically with time $c_i(0, t) = \beta_i \sqrt{t}$:

$$c_i(x, t) = \sum_{j=1}^n \frac{\Delta_j^n}{\det(k_j^n)} \left\{ \sqrt{\frac{t}{\pi}} \exp\left[-\frac{x^2 (k_j^n)^2}{4t}\right] - \frac{xx^j}{2} \operatorname{erfc}\left(\frac{xx^j}{2\sqrt{t}}\right) \right\},$$

where $\Delta_j^n = 2\sqrt{p}\Delta_j$ and does not contain p .

If $c_i(x, 0) = c_i^0$, then in the solution c_i should be replaced by $(c_i - c_i^0)$.

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USING THE HEAT-BALANCE INTEGRAL FOR THE SOLUTION OF CERTAIN SOIL FREEZING PROBLEMS

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The article presents an analytical method by which one can determine the maximum depth of a cooled soil zone and the thermal influence factor, the latter being equal to the ratio of the maximum cooling depth to the depth of the ice zone produced by artificial freezing of a soil.

The formula for calculating the depth of a cooled soil zone $X(\tau)$ is

$$X(\tau) = 3.46 \sqrt{a_2 \tau}, \quad (1)$$

where a_2 denotes the thermal diffusivity of soil in the cooled zone and τ denotes the freezing time.

In order to calculate the thermal influence factor, one first determines the proportionality factor β from the characteristic equation of the Stefan solution [1].

The thermal influence factor can be determined from the formula

$$m = \frac{3.46 \sqrt{a_2}}{\beta}. \quad (2)$$

Solutions are obtained here by using the thermal balance integral [2].

Calculated values of thermal influence factors are shown for sandy soils with a varying moisture content W at the inherent soil temperature T_0 and the freezing temperature T_f .

An analysis of the calculated results shows that in some cases the magnitude of the thermal influence factor changes during freezing and that, therefore, it may not be taken on the basis of averaged data [3, 4] but must be first calculated analytically for each case.

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TEMPERATURE FIELD OF A TOOL SUBJECTED TO THERMAL FLUXES OF A COMPLEX PATTERN IN AN UNSTABLE MODE

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During a stamping operation the engraving on the tool is repeatedly subjected to thermal fluxes of diverse physical characteristics and of varying intensity. In order to simplify the mathematical solution of the heat problem, it is worthwhile to represent the variable flux during every τ_i long stage of an arbitrary τ_c long cycle by piecewise constant functions.

The solution to the heating cycle problem is illustrated on a thick-walled flat punch of thickness L , whose engraved surface ($x = 0$) is subjected successively to fluxes q_i^+ ($i = 1, 2, \dots, m$) while a heat transfer is taking place between its outside surface ($x = L$) and the ambient medium (H is the heat-transfer coefficient). The solution for the temperature field of the tool during the k -th stage of an arbitrary r -th cycle at time t_{kc} ($0 \leq t_{kc} \leq \tau_k$) is

$$T_k^r(x, t_{kc}) = T_c(x, t_{kc}) + \sum_{l=1}^{k-1} T_l^r(x, t_{kc}) + \sum_{j=k}^m T_j^r(x, t_{kc}), \quad (1)$$

where

$$T_c(x, t_{kc}) = \frac{q_k^+}{\lambda} \left\{ \frac{\lambda + (L-x)H}{H} - 2 \sum_{n=1}^{\infty} C_n(x) \exp[\kappa u_n^2 (-t_{kc})] \right\},$$

$$T_l^r(x, t_{kc}) = -2 \frac{q_l^+}{\lambda} \sum_{n=1}^{\infty} \frac{C_n(x)}{A_n} \left\{ \exp[(-\kappa u_n^2)((r-1)\tau_c + t_{kc})] - \exp[\kappa u_n^2(\tau_c - t_{kc})] \right\} \left[\exp\left(-\kappa u_n^2 \sum_{j=l}^{k-1} \tau_j\right) - \exp\left(-\kappa u_n^2 \sum_{j=l+1}^{k-1} \tau_j\right) \right] \quad (l = 1, 2, \dots, k-1),$$

$$T_j^r(x, t_{kc}) = -2 \frac{q_j^+}{\lambda} \sum_{n=1}^{\infty} \frac{C_n(x)}{A_n} \left\{ \exp[(-\kappa u_n^2)((r-1)\tau_c + t_{kc})] - \exp(-\kappa u_n^2 t_{kc}) \right\} \left[\exp\left(\kappa u_n^2 \sum_{i=k}^{j-1} \tau_i\right) - \exp\left(\kappa u_n^2 \sum_{i=k}^j \tau_i\right) \right] \quad (j = k, k+1, \dots, m),$$

$$C_n(x) = \frac{\lambda u_n \cos[u_n(L-x)] + H \sin[u_n(L-x)]}{u_n^2 [\lambda L u_n \cos u_n L + (\lambda + HL) \sin u_n L]},$$

$$A_n = 1 - \exp(\kappa u_n^2 \tau_c),$$

and u_n are the roots of the equation $\tan uL = H/\lambda u$.

Here λ and κ are the thermal conductivity and the thermal diffusivity. The solution for the quasi-steady state ($r \rightarrow \infty$) is easily obtained from (1).

With the transient and the quasisteady solutions on hand, it is possible to make an upper-limit estimate (r^+ cycles) of the transition boundary between one stage and the next.

The obtained solutions are also useful for an analytical determination of the thermal fluxes prevalent during each stage. Flux q_i^+ turns out to be a function of the tool surface temperature before the beginning of the i -th stage:

$$q_i^+ = \varphi_i [T_i(x=0; t_{ic}=0)] = \varphi_i(T_{fi}). \quad (2)$$

After this function has been expanded into a Taylor series near the characteristic point T_{0i} , the first two terms of this series alone yield

$$\varphi_i(T_{fi}) = a_i T_{fi} + b_i. \quad (3)$$

The determination of fluxes reduces to a solution of the system

$$\sum_{i=1}^m (\delta_{ij} - a_j) \omega_i(0, 0) q_i^+ = b_j \quad (j = 1, 2, \dots, m), \quad (4)$$

where δ_{ij} is the Kronecker delta and ω_i are cofactor functions to q_i^+ in the solution to (1). The solution to system (4) is written as

$$q_i^+ = \sum_{j=1}^m \gamma_{ji} b_j, \quad (5)$$

where γ_{ji} are elements of the inverse matrix with respect to which the matrix of coefficients $C_{ji} = (\delta_{ji} - a_j) \omega_i(0, 0)$ is the direct one. The numerical values of fluxes and the temperature field agree closely with experimental data obtained in stamping large drawn container bottoms.

A wide range of stamping operations is characterized by an instability of individual stages within a single cycle. The solution obtained for such cases is the mathematical expectation of the temperature field of the tool during the quasisteady stage, which takes into account the basic statistical characteristics of unstable processes: the mathematical expectation of the stamping parameters and their dispersions. The obtained results are useful for analyzing various technological variants of the stamping process in the design stage, for evaluating the temperature field, and for evaluating the quality of forgings.

ACCURACY OF REPLACING A CYLINDRICAL HEAT SOURCE BY A POINT SOURCE IN THE DETERMINATION OF THE TEMPERATURE FIELD OF A SEMIINFINITE BODY

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UDC 536.24

In order to correctly design heating systems whose elements are installed in ground floors, it is necessary to solve the heat transfer and temperature field problem for a series of ducts laid in a semi-infinite base.

A theoretical solution to this problem has been obtained by A. I. Ioffe by the source-sink and superposition method as well as by A. A. Sander by the conformal mapping method. In the resulting relations a cylindrical heat source is replaced by a point source. The purpose of this author's study is to evaluate the error in those calculations.

On the basis of the solutions to the system of equations describing the temperature field in a semi-infinite body, isothermal lines and diagrams are plotted for determining the magnitude of the correction ε to be put in the equations for more accurate thermal engineering calculations and applications.

The correction ε to the Sander equation is obtained as the ratio of two lengths: the circumference of a duct with a given diameter and the length of the isothermal line plotted for $t_{x,y} = t_{ds}$ ($t_{x,y}$ denotes the temperature at a point in space coordinates and t_{ds} denotes the duct surface temperature).

The correction to the Ioffe equation is obtained as the ratio of two thermal resistances: the thermal resistance of the base in which a duct of a given diameter is laid to a given depth and the thermal resistance of a base in which a duct with a circumference equal to the length of the isothermal line for $t_{x,y} = t_{ds}$ is laid to a new depth.

An analysis of the obtained and graphically evaluated results shows that, at a relatively shallow depth of ducts in a base and at a relatively close spacing of ducts, as is the case in panel heating systems, the

quantity of heat according to the equations is somewhat less than the quantity of heat actually emitted from ducts. The error in using the equations increases with the depth of duct laying and with the duct diameter.

DETERMINING THE HEAT RESISTANCE OF CYLINDRICAL SPECIMENS BY THE AXIAL HEAT-FLUX METHOD

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The axial heat-flux method of determining the heat resistance of brittle materials is corroborated here on cylindrical specimens. In one variant of this method a sufficiently long solid cylinder is heated at one end by a constant thermal flux from a plasma or electron-beam torch. The transient temperature field is assumed to be varying only along the cylinder height and, according to the integral heat-balance method [1], is approximated by a cubic parabola. The error of the approximate solution to the equation of heat conduction in the space-time $z/\sqrt{a\tau} \leq 1.7$ does not exceed 5%. The approximate solution to the decoupled axisymmetrical thermoelasticity problem in the quasistatic mode is obtained for a semiinfinite cylinder, on the basis of known relations for temperature stresses in an infinitely long cylinder with an arbitrary heightwise temperature distribution [2]. This solution is expressed in a closed form in terms of the temperature field and so-called temperature influence functions, i.e., in terms of temperature stresses in a unit-step temperature field. By a simple assumption, using a definite even-order form of the influence functions [2], it is possible to satisfy rigorously one of the boundary conditions concerning either axial or tangential stresses at the end surface of the semiinfinite cylinder. The other boundary condition (concerning the other of the two stresses) at the end surface is satisfied in the St. Venant sense. Stress field calculations are made here for a cylinder heated at the end surface by a constant heat flux while the boundary condition concerning the axial stresses is satisfied at that surface. It is shown that the maximum axial stress

$$\sigma_{z\max} \approx 0.119\alpha E \frac{qR}{\lambda} \quad (1)$$

for $\mu = 0.3$ occurs in the tensile zone along the cylinder axis at the point $z_{\max} \approx 0.78R$ at the time $\tau_{\max} \approx 0.26R^2/\alpha$.

With the other boundary condition (concerning the tangential stresses) at the end surface is satisfied approximately [3], which allows one to estimate the accuracy of the obtained results, the magnitude of σ_{\max} (1) varies by not more than 20%. This method was tried with aluminum oxide and zirconium carbide specimens heated with a plasma torch, yielding a close agreement between tested and calculated values pertaining to the fracture surface coordinate of these specimens.

NOTATION

α	is the linear thermal expansivity;
E	is the modulus of normal elasticity;
μ	is the Poisson ratio;
λ, a	are the thermal conductivity and thermal diffusivity;
q	is the thermal flux density;
R	is the radius of cylinder.

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THERMOELASTIC STRESSES INDUCED IN BODIES OF
SIMPLE SHAPES WHEN HEATED IN THE PARALLEL-FLOW
AND THE COUNTERFLOW MODE *

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UDC 536.5

Cold billets are, as a rule, heated up in apparatus operating with parallel and counterflow. It is of interest here to determine the magnitude of thermoelastic stresses and the trend of their variation.

Formulas are derived for calculating the thermoelastic stresses induced in an infinitely long cylinder and an infinitely large plate by heating in parallel-flow and counterflow furnaces. These formulas are presented in the critical form:

for a plate

$$\frac{\sigma(1-\nu)}{\beta E (t_m^0 - t_g)} = f(\text{Bi}, \text{Fo}, W);$$

and for a cylinder

$$\begin{aligned} \frac{\sigma_r(1-\nu)}{\beta E (t_m^0 - t_g)} &= f_r(\text{Bi}, \text{Fo}, W), \\ \frac{\sigma_\theta(1-\nu)}{\beta E (t_m^0 - t_g)} &= f_\theta(\text{Bi}, \text{Fo}, W), \\ \frac{\sigma_z(1-\nu)}{\beta E (t_m^0 - t_g)} &= f_z(\text{Bi}, \text{Fo}, W), \end{aligned}$$

where σ is the stress, kg/mm²; ν is the Poisson ratio; β is the linear thermal expansivity, deg⁻¹; E is the modulus of normal elasticity, kg/mm²; t_m^0 is the mean initial temperature of the metal, °C; t_g is the gas temperature, °C; Bi is the Biot number; Fo is the Fourier number; and W is the ratio of specific heats of the heated body and the gas, respectively.

The solutions are presented graphically, in the form of curves for convenient practical use. The applicability of the derived relations is illustrated on an example.

CALCULATING THE TEMPERATURE OF A MEDIUM
INSIDE A CLOSED SPACE WHEN THE OUTSIDE
AMBIENT TEMPERATURE VARIES PERIODICALLY †

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UDC 536.2.01

The temperature t_{in} of a medium inside a closed shell of low-thermal conductivity can be determined, if it is assumed constant over the entire volume at every instant of time, from the heat-balance equation:

$$k \frac{dt_{in}(\tau)}{d\tau} + t_{in}(\tau) = t(0, \tau),$$

where $k = V_{in}c_{in}/\alpha_{in}F_{in}$.

* Institute of Metallurgy, Dnepropetrovsk. Original article submitted November 26, 1970; abstract submitted June 2, 1971.

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Assuming that the shell is very thin as compared to its other dimensions, $t(0, \tau)$ can be determined from the solution to the equation of heat conduction in an infinitely large flat plate with boundary conditions of the third kind – which correspond to the said heat-balance equation for the inside surface of the shell and to a temperature variation $t_{\text{out}} = \theta_S + t_M \cos \omega\tau$ at the outside surface. When the periodic temperature variation of the outside medium is more complex, it can be expressed as the sum of harmonic components and each component can be considered separately.

The solution to this equation has been obtained with the aid of a Laplace transformation, first for $t(x, \tau)$ and then, letting $x = 0$ in the latter, for $t(0, \tau)$. After insertion of the thus determined function $t(0, \tau)$ into the heat-balance equation, the value of $t_{\text{in}}(\tau)$ is found as the solution to a first-order linear differential equation. It is

$$t_{\text{in}}(\tau) = \theta_S + (t_0 - \theta_S) \exp\left(-\frac{\tau}{k}\right) + t_M \frac{\sqrt{N_i N_{-i}}}{\omega^2 k^2 + 1} \\ \times \left\{ [\cos(\omega\tau - \varphi) + \omega k \sin(\omega\tau - \varphi)] - [\cos \varphi - \omega k \sin \varphi] \exp\left(-\frac{\tau}{k}\right) \right\} \\ + \sum_{n=1}^{\infty} A_n (\mu_n \text{Bi}_2 \text{Fo}^*)^{-1} \left[t_0 - \theta_S - t_M \frac{\mu_n^2}{\mu_n^4 + \text{Pd}^2} \right] \left[\exp(-\mu_n^2 \text{Fo}) - \exp\left(-\frac{\text{Fo}}{\text{Fo}^*}\right) \right],$$

where μ_n are the roots of the characteristic equation

$$\text{tg}(\mu + \beta) = \frac{1 - \text{Fo}^* \mu^2}{\mu \text{Bi}_2 \text{Fo}^*},$$

with $\text{Fo}^* = \alpha k / \delta^2$ and $\tan \beta = \mu \text{Bi}_1$.

In practice it is common that $\exp(-\text{Fo}/\text{Fo}^*)$ tends to zero very fast while Bi_1 is very large at the outside surface. Under these conditions the solution becomes much simpler and the characteristic equation is, by means of a certain substitution, transformed into an equation which has been proposed by M. D. Mikhaikov and whose roots have been tabulated precisely. The given equation is recommended for calculating the thermal insulation and the time in which the temperature of the inside medium will reach a prescribed level: in a thermal analysis of isothermally heated cars, in engineering thermophysics, in the design of components for thermopower stations, etc.

ANALYTICAL SOLUTION OF THE PROBLEM CONCERNING TRANSIENT INTERACTION BETWEEN NATURAL AND FORCED CONVECTION IN A RECTANGULAR DUCT

V. A. Dubrovik and V. P. Kharitonov

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The article presents an analysis of the transient simultaneous natural and forced convection in a vertical rectangular duct. It covers the fully developed flow and heat transfer during a linear variation of temperature along the duct. First of all, a prevailing steady-state convection is perturbed by an arbitrary time-variation of the pressure gradient and the wall temperature around the duct perimeter. For the conditions considered here, the transient flow and heat transfer are described by a system of two linear partial differential equations in dimensionless variables. Both the velocity and the temperature distribution across a duct section are sought in the form of binary sine series and then, by means of a Laplace transformation, the solution is found to the found system of ordinary differential equations.

Tomsk Polytechnic Institute. Original article submitted January 13, 1971; abstract submitted June 24, 1971.

The transient process which occurs as a result of a stepwise change in the wall temperature or in the pressure gradient is analyzed thoroughly. It appears in this case that at large positive Rayleigh numbers $Ra \gg 0$ (which correspond to downward heating) and at a Prandtl number $Pr = 1$ both velocity and temperature variations are damped-oscillatory with a period proportional to $1/\sqrt{Ra}$, while at negative Rayleigh numbers $Ra \leq 0$ and any Prandtl number these variations are overdamped. Furthermore, in a rectangular duct steady-state is reached faster than in a circular duct, while large velocity and temperature oscillations begin at much higher Rayleigh numbers than in a circular duct.

These results are useful for the calculation, design, and selection of active heat-exchanger components.

IDENTIFICATION OF THERMOCHEMICAL INTERACTION PROCESSES

E. S. Lobova and S. A. Malyi

UDC 62-50:669

The solution to problems concerning the optimal control of thermochemical interaction becomes simplified, in principle, when the thermochemical interaction process can be successfully described by a differential equation

$$\frac{dw}{d\tau} = \frac{F(u)}{W(w)}, \quad (1)$$

where w measures the quantity of material used up in the reaction; τ is the time; u is a vector which characterizes the thermochemical interaction conditions (e.g., $u = (t, \alpha, p, \dots)$, with α denoting the potential of the medium with which the metal interacts; t denoting the temperature of the reacting surface; and p denoting the pressure); $F(u)$ is a function which defines the effect of conditions u (under which the thermochemical interaction occurs) on the rate of this interaction; and $W(w)$ is a function which defines the thermochemical interaction as a function of time [1].

If the thermochemical interaction is adequately well described by Eq. (1), then the same solution is obtained to the optimal control problem regardless of what function of time the thermochemical interaction is. This is equivalent to stating that an optimal process is technically feasible at all, even though function $W(w)$ varies in an uncontrollable manner in the course of the real process [2].

The Butkovskii-Sun-Tsian method [3] is the best one to use for fitting test data into Eq. (1), to yield

$$\min_{a, W(w), F(u)} \iint_{\Omega} \left[f(w, u) - a \frac{F(u)}{W(w)} \right]^2 dw du.$$

In the region of test data $dw/d\tau = f(w, u)$, with $f(w, u)$ given in the tables.

If n and m are the numbers of subintervals along the w and the u axis, respectively, while W_p , F_q , and f_{pq} are the values of the function at points p and q , respectively, then the problem reduces to seeking the maximum eigenvalue $\lambda_{\max} = a^2$ and the corresponding eigenfunctions $F_q(u)$, $W_p^{-1}(w)$ of the system

$$\lambda F_q = \sum_{i=1}^m \sum_{p=1}^n f_{pq} f_{pi} F_i, \quad q = 1, 2, \dots, m, \quad (2)$$

$$\lambda W_p^{-1} = \sum_{i=1}^n \sum_{q=1}^m f_{pq} f_{iq} W_i^{-1}, \quad p = 1, 2, \dots, n. \quad (3)$$

Institute of Control Problems (of Automation and Telemechanics), Moscow. Original article submitted February 15, 1971; abstract submitted August 23, 1971.

In the article are shown calculated results which indicate that the degree of approximation of test data for grades 16GNMA and Kh18N9 steel by formula (1) is entirely satisfactory and, consequently, the problem of minimizing the oxidation of steel may be considered solved.

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ANALYSIS OF THE THREE-DIMENSIONAL TEMPERATURE FIELD IN A FINNED PANEL WITH THE AID OF A SMALL-GRID MODEL

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UDC 536.24.001.57

A procedure is described here for solving steady-state three-dimensional heat-conduction problems on a small MSM-1 grid model by adding resistors made of electrically conductive paper. In constructing a grid model, which consists of R_λ resistors distributed along the x , y , z coordinates and R_α resistors simulating the heat transfer to the ambient medium, resistors distributed in planes $z = 0$, $z = \Delta z$, $z = 2\Delta z$, . . . are laid out on the MSM-1 grid. The resulting two-dimensional resistor networks for the three-dimensional model are interconnected, respectively, through the MSM-1 panel on which the boundary conditions are stipulated.

The resistors which interconnect the nodes of the various sections are, as also the R_α resistors, made of electrically conductive paper strips. The choice of this type of resistors is dictated by the low cost and easy assembly.

By this method one can simulate the three-dimensional temperature field in a finned panel of heavy-grade fireproof concrete insulated thermally with a diatomaceous brick lining. The results of such a simulation agree fairly well with test data.

AN ELECTRICAL TRANSMISSION LINE - AN ANALOG
OF TRANSIENT HEAT CONDUCTION IN PLANE,
CYLINDRICAL, AND SPHERICAL BODIES

P. A. Voronin

UDC 536.24.001:681.142.353

The differential thermal parameters are derived for plane, cylindrical, and spherical bodies: the thermal resistance $R_{O.T}$ and the thermal capacitance $C_{O.T}$ per unit length of the reference dimension b (thickness or radius of the body). The differential parameters for a flat body of height h (tape, busbar strip) do not depend on its thickness, nor on its length (y):

$$R_{O.T.P} = \frac{dR_T(y)}{dy} = \frac{1}{\lambda hl}; \quad C_{O.T.P} = \frac{dC_T(y)}{dy} = c_y \rho hl.$$

When a cylindrical body of length $l \gg b$ is heated uniformly at the lateral surface, its differential thermal resistance and differential thermal capacitance are functions of the heat-propagation coordinates:

$$R_{O.T.P} = \frac{1}{2\pi\lambda l(b-y)}; \quad C_{O.T.P} = 2\pi c_y \rho l(b-y).$$

When a spherical body is heated uniformly from all directions, its differential thermal resistance and differential thermal capacitance are

$$R_{O.T.S} = \frac{1}{4\pi\lambda(b-y)^2}; \quad C_{O.T.S} = 4\pi c_y \rho(b-y)^3.$$

In these formulas ρ is the density of the material; c_y is its specific heat; and λ is its thermal conductivity.

The correspondence is shown between these parameters and distributed longitudinal and transverse parameters of an electrical transmission line, namely its differential resistance and differential capacitance. Moreover, the voltage corresponds to the temperature and the electric current corresponds to the thermal flux. Also given are the ratios of the similarity scale factors for simulating thermal processes in flat bodies by homogeneous noninductive electrical transmission lines with distributed parameters and thermal processes in cylindrical or spherical bodies by nonhomogeneous such lines. The basic ratios are

$$\frac{m_T}{m_F m_R m_y} = 1; \quad \frac{m_T m_C m_y}{m_F m_t} = 1.$$

The generalized ratio of scale factors characterizing the design parameters of a noninductive line is

$$\frac{m_R m_C m_y^2}{m_t} = 1.$$

Here m_T is the temperature scale; m_F is the thermal flux scale; m_R is the thermal resistance scale; m_C is the thermal capacitance scale; m_t is the time scale; and m_y is the length scale.

In practice a noninductive line may be replaced by its electrical model, namely an RC-network consisting of Π -fourpoles with resistances in the line branches and capacitances in the shunting branches. The parameters of such electrical fourpoles and their thermal analogs are derived in this article.

EFFECT OF THE THERMODYNAMIC PROPERTIES OF
CRYOGENIC LIQUIDS ON THEIR PITTING ACTION
DURING CAVITATION

Yu. E. Krot and V. I. Grushko

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Solid surfaces become pitted (eroded) as a result of implosion of cavities in the liquid near such surfaces. If an implosion of cavities occurs adiabatically, then the vapor contained in them superheats and becomes a gaseous shock absorber which inhibits further implosions. A hypothesis has been proposed that, owing to the damping action of vapors, cryogenic liquids whose vapors are very expansive will have a lesser pitting effect on container materials than cryogenic liquids whose vapors are much less expansive.

This hypothesis was tested on a copper specimen at the temperature of 77°K exposed to the following cryogenic liquids: oxygen ($P_{\text{vap}} = 151$ mm Hg), natural gas ($P_{\text{vap}} \cong 400$ mm Hg), and nitrogen ($P_{\text{vap}} = 760$ mm Hg). The copper was pitted most in oxygen, less in natural gas, and least in nitrogen.

The degree of damping by a vapor is determined by the thermodynamic properties of the cavitating liquid and can be evaluated in terms of the thermodynamic parameter B_{eff} [1]:

$$B_{\text{eff}} = \left(\frac{\rho_L c_L \Delta T}{\rho_V L} \right)^2 \frac{K_L}{R_0} \left(\frac{\rho_L}{\Delta P} \right)^{1/2}$$

The values of B_{eff} calculated for several liquids at various temperatures are shown in Fig. 1. The dashed line, which corresponds to $B_{\text{eff}} = 10^3$, divides the cavitation range into one where cavitation depends on the damping action of the vapor and one where it almost does not.

NOTATION

ρ_L	is the density of liquid;
ρ_V	is the density of vapor;
c_L	is the specific heat of liquid;
ΔP	is the drop in local pressure;
ΔT	is the drop in the vapor-liquid equilibrium temperature corresponding to a pressure drop ΔP ;
L	is the latent heat of evaporation;
$K_L = \lambda_L / \rho_L c_L$;	
λ_L	is the thermal conductivity of liquid;
R_0	is the equilibrium radius of cavitation bubble.

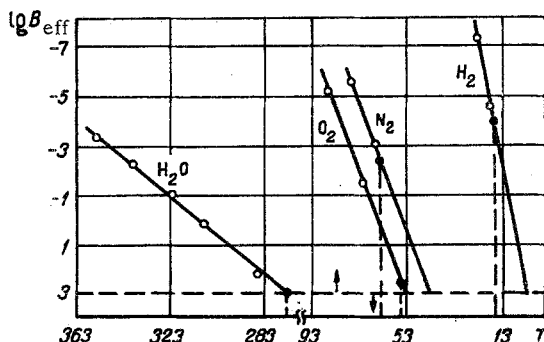


Fig. 1. Temperature-dependence of the thermodynamic parameter, for cryogenic liquids and for water (black dots correspond to the freezing temperatures). Temperature T , °K.

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Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Khar'kov. Original article submitted May 20, 1971; abstract submitted August 9, 1971.